On the twist-3 contribution to h_L in the instanton vacuum

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Abstract

We show that the instanton model of the QCD vacuum indicates the parametric smallness of the twist-3 contributions to the polarized structure function h_L. This smallness is related to the diluteness of the QCD instanton vacuum.

1. Among higher twist parton distributions the twist-3 distributions play a special role. In many physical observables the twist-3 distributions enter not suppressed by powers of the hard scale relative to twist-2 distributions. Therefore the determination of twist-3 distributions does not encounter the conceptual problems of the separation of power suppressed contributions from those that are suppressed by only logarithms. Examples of such observables are spin asymmetries in DIS on transversely polarized targets (g_2) [1] and single spin azimuthal asymmetries in semi-inclusive production of hadrons (h_L) [2, 3, 4]. The experimental data of DIS on transversely polarized targets have already reached the precision to estimate the twist-3 contributions to the observables, see recent measurements by E155 collaboration [6]. Recent HERMES and SMC data on single spin azimuthal asymmetries [5, 7] provide the possibility to estimate the quark transversity distribution h_1 in the nucleon if, among other things, one would be able to estimate the twist-3 contribution to h_L. The objective of this report is to make an estimate of the size of the twist-3 contribution to h_L in the instanton model of the QCD vacuum.

The twist-3 distributions are given by nucleon matrix elements of mixed quark-gluon operators. These matrix elements are very sensitive to the correlations of non-perturbative gluon and quark fluctuations in the QCD vacuum. The theory of such fluctuations is provided by the instanton model of the QCD vacuum [8] (for a review see [9, 10]). A nice feature of the instanton model of the QCD vacuum is the existence of a small parameter– the ratio of the average instanton size $\bar{\rho}$ to the average distance between instantons $\bar{R}$ ($\bar{\rho}/\bar{R} \approx 1/3$). This parameter was first anticipated in ref. [11] from phenomenological considerations, obtained in dynamical calculations of [12] and recently confirmed by direct measurements on the lattice [13].

In ref. [14] a method was developed to calculate hadronic matrix elements of mixed quark gluon operators in the instanton vacuum. Later this method was applied to estimates of higher twist operators [15]. In particular it was shown that the twist-3 contribution to the structure function g_2 is parametrically small relative to twist-2 and twist-4 contributions. For example, the third moment of g_2

$$\int_0^1 dx\, x^2 g_2(x, Q^2) \;=\; -\frac{1}{3}a^{(2)} \;+\; \frac{1}{3}d^{(2)} \;+\; O\left(\frac{1}{Q^2}\right) \tag{1}$$

can be splitted into the twist-2 part $a^{(2)}$ and twist-3 part $d^{(2)}$. In the instanton vacuum, the twist-2 part is parametrically of order

$$a^{(2)} \sim (\bar{\rho}^2/\bar{R}^2)^0 \sim 1 \tag{2}$$

whereas the twist-3 part behaves like

$$d^{(2)} \sim (\bar{\rho}^2/\bar{R}^2)^2 \log(\bar{\rho}^2/\bar{R}^2) \sim 10^{-3} \tag{3}$$

(see [15] for details). This strong suppression of the twist-3 contribution relative to twist-2 one is related to the specific spin-colour structure of the instanton field and its properties under conformal transformations. Using this fact one can conclude that the suppression of the twist-3 part persists also for higher moments, not only for the lowest one. Here we repeat the analysis for the lowest moment of h_L^{tw3}, leaving the general proof for a comprehensive paper [16].

2. The Mellin moments of $h_L(x)$ can be splitted into twist-2 and twist-3 part [17]

$$\mathcal{M}_n[h_L] \equiv \int_0^1 dx \ x^n \ h_L(x) = \frac{2}{n+2}\mathcal{M}_n[h_1] + \mathcal{M}_n[h_L^{tw3}] , \tag{4}$$

where the first term is related to the Mellin moment of the twist-2 transversity quark distribution $h_1(x)$ [19]. The moments of h_L^{tw3} are related to the following matrix elements of mixed quark-gluon operators [17]:

$$\mathcal{M}_n[h_L^{tw3}] = \sum_{l=2}^{\frac{n+1}{2}} \left(1 - \frac{2l}{n+2}\right) b_{nl}(\mu^2) \tag{5}$$

with

$$\langle pS|R_{nl}^{\mu_1\cdots\mu_n}(\mu^2)|pS\rangle = 2b_{nl}(\mu^2)M^2(S^{\mu_1}P^{\mu_2}\ldots P^{\mu_n} - \text{traces}) \tag{6}$$

The general form of the operators $R_{nl}^{\mu_1\cdots\mu_n}(\mu^2)$ can be found in [17].

Here we shall be interested in the lowest non-vanishing moment $n = 3$

$$\mathcal{M}_3[h_L^3] = \frac{1}{5}b_{32}(\mu^2) \tag{7}$$

with $b_{32}(\mu^2)$ defined through the matrix element

$$\langle PS|R_{32}^{\delta\alpha\beta}(\mu^2)|PS\rangle = 2b_{32}(\mu^2)M^2\mathcal{S}(S^\delta P^\alpha P^\beta - \text{traces}) . \tag{8}$$

where $\mathcal{S}$ denotes the symmetrization of Lorentz indicies and the local operator has the form

$$R_{32}^{\delta\alpha\beta} = \frac{1}{2}\mathcal{S}\bar{\psi}\sigma^{\gamma\delta}i\gamma_5[i\nabla^\alpha, iF_\gamma^\beta]\psi - \text{traces} \tag{9}$$

or equivalently

$$R_{32}^{\delta\alpha\beta} = -\frac{i}{2}\mathcal{S}\bar{\psi}\sigma^{\gamma\delta}\gamma_5(D^{\alpha ac}F_\gamma^{\beta\,c})\frac{\lambda^a}{2}\psi - \text{traces} . \tag{10}$$

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We shall compute the matrix element (8) in the instanton model of the QCD vacuum using the technique of refs. [14, 15].

The effective low–energy theory one derives from the instanton vacuum is formulated in terms of degrees of freedom which are pions (Goldstone bosons) and massive "constituent" quarks. It is described by the effective action [8, 9]

$$S_{\text{eff}} = \int d^4x \, \bar{\psi}(x) \left[i\gamma^\mu \partial_\mu - MF(\overleftarrow{\partial}) e^{i\gamma_5 \tau^a \pi^a(x)} F(\overrightarrow{\partial}) \right] \psi(x). \tag{11}$$

Here, M is the dynamical quark mass generated by the spontaneous breaking of chiral symmetry; parametrically it is of order:

$$M\bar{\rho} \sim \left(\frac{\bar{\rho}}{\bar{R}} \right)^2, \tag{12}$$

and $F(k)$ is a form factor proportional to the wave function of the instanton zero mode, which drops to zero for momenta of order $k \sim \bar{\rho}^{-1}$. Mesonic correlation functions computed either with the effective action, Eq.(11), using the $1/N_c$–expansion [8] or by more elaborate numerical simulations [10] show excellent agreement with phenomenology.

In order to find the parametric behaviour of b_{32} in the packing fraction $\bar{\rho}^2/\bar{R}^2$ it is enough to compute the matrix element (8) in constituent quark states. In order to accomplish this one has to transform the operator (10) into the corresponding effective operator in the effective low-energy theory (11). The details of such transformation can be found in [14, 15]. Here we only report the main technical steps.

First we compute the covariant derivative of the gluon field strength on the field of one instanton (anti-instanton) $I(\bar{I})$:

$$D_\alpha^{ac} F_{\beta\gamma}^c(x)_{I(\bar{I})}$$
$$= (\eta^{\mp})_{\lambda\rho}^a \frac{-48\bar{\rho}^2}{(x^2 + \bar{\rho}^2)^3} \left[\left(\frac{x_\alpha x_\beta x_\lambda}{x^2} - \frac{1}{6}(x_\alpha \delta_{\beta\lambda} + x_\beta \delta_{\alpha\lambda} + x_\lambda \delta_{\alpha\beta}) \right) \delta_{\gamma\rho} - (\beta \leftrightarrow \gamma) \right]. \tag{13}$$

Its Fourier transform has the form

$$\mathcal{K}_{\lambda\rho\alpha\beta\gamma}(k) = i\bar{\rho}^2 \mathcal{K}(k^2) \left[\left(\frac{k_\alpha k_\beta k_\lambda}{k^2} - \frac{1}{6}(k_\alpha \delta_{\beta\lambda} + k_\beta \delta_{\alpha\lambda} + k_\lambda \delta_{\alpha\beta}) \right) \delta_{\gamma\rho} - (\beta \leftrightarrow \gamma) \right], \tag{14}$$

where

$$\mathcal{K}(k^2) = (24\pi)^2 \left[-\frac{16}{t^6} + \left(\frac{16}{t^5} + \frac{4}{t^3} + \frac{1}{4t} \right) K_1(t) + \left(\frac{8}{t^4} + \frac{1}{t^2} + \frac{1}{24} \right) K_0(t) \right], \quad t = |k|\bar{\rho}. \tag{15}$$

Here $K_\nu(t)$ are modified Bessel functions. Using this result we can easily derive the form of the effective operator[1]:

$$R_{\delta\alpha\beta}^{\text{eff}}(x) = \frac{M}{N_c} \int d^4z \, \mathcal{K}_{\lambda\rho\alpha\beta\gamma}(x - z)$$
$$\times \psi^\dagger(x) \sigma_{\gamma\delta} \gamma_5 \frac{\lambda^a}{2} \psi(x) \psi^\dagger(z) \frac{\lambda^a}{2} \sigma_{\lambda\rho} \frac{1 \pm \gamma_5}{2} \psi(z). \tag{16}$$

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[1] We give the expression for the case of one flavour which is enough to compute the matrix element of the operator between constituent quark states

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Now it is easy to compute the matrix element of this operator in constituent quark states

$$\langle p|R_{\delta\alpha\beta}^{\text{eff}}|p\rangle = \frac{iM}{2}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\mathcal{K}_{\lambda\rho\alpha\beta\gamma}(k)$$

$$\times\left[\frac{F(p)F(p-k)}{(p-k)^2+M^2F^4(p-k)}\text{Tr}\Big[\Lambda_{p,S}\sigma_{\gamma\delta}\gamma_5\Big(\not{p}-\not{k}+iMF^2(p-k)\Big)\sigma_{\lambda\rho}\frac{1\pm\gamma_5}{2}\Big]\right.$$

$$\left.+\frac{F(p)F(p+k)}{(p+k)^2+M^2F^4(p+k)}\text{Tr}\Big[\Lambda_{p,S}\sigma_{\lambda\rho}\frac{1\pm\gamma_5}{2}\Big(\not{p}+\not{k}+iMF^2(p+k)\Big)\sigma_{\gamma\delta}\gamma_5\Big]\right],\quad(17)$$

where the projector on quark states with definite momentum and polarization vector has the form

$$\Lambda_{p,S}=u(p,S)\bar{u}(p,S)=\frac{-i\not{p}+M}{2}\Big(1+i\gamma_5\not{S}\Big).\qquad(18)$$

The traceless part of the operator $R_{\delta\alpha\beta}^{\text{eff}}$ which is related to b_{32} can be isolated by contracting the Lorentz indices $\delta\alpha\beta$ with a light-cone vector n, such that $n\cdot p$ and $n\cdot S$ are non-zero:

$$n_\alpha n_\beta n_\delta\langle p|R_{\delta\alpha\beta}^{\text{eff}}|p\rangle=2M^2I(p)(n\cdot p)^2(n\cdot S).\qquad(19)$$

The quantity b_{32} is related to $I(p)$ as $b_{32}=-I(M)$. The expression for $I(p)$ is given by a simple integral:

$$I(p)=\bar{\rho}^2\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{F(p)F^3(p-k)\mathcal{K}(k^2)}{(p-k)^2+M^2F^4(p-k)}\left[\frac{k\cdot p}{p^2}-2\frac{(k\cdot p)^3}{k^2p^4}\right].\qquad(20)$$

Its small p-behaviour has obviously the form [2]:

$$I(p)\sim p^2\bar{\rho}^2\ln(|p|\bar{\rho}).\qquad(21)$$

From this we conclude that b_{32} is parametrically suppressed by the packing fraction of the instanton liquid

$$b_{23}\sim(\bar{\rho}^2/\bar{R}^2)^2\log(\bar{\rho}^2/\bar{R}^2),\qquad(22)$$

i.e. as for twist-3 contribution to $g_2(x)$, see eq. (3). This is the main result of this report. We can expect that the twist-3 part of h_L is also numerically much smaller than its twist-2 part because the twist-2 part of h_L behaves in the packing fraction as $\sim(\bar{\rho}^2/\bar{R}^2)^0$ [20]. The obtained suppression of the twist-3 part of h_L refers to a low normalization point of order $\sim 1/\bar{\rho}\approx 0.6$ GeV. Under evolution to higher normalization points the twist-3 part of h_L^{tw3} dies out faster than h_L^{tw2} [18], so that the suppression of h_L^{tw3} relative to h_L^{tw2} will be even more pronounced at higher Q^2.

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[2]The integral (20) has generic form (for $n=3$):

$$I_n(p)\propto\bar{\rho}^2\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{F(p)F^3(p-k)\mathcal{K}(k^2)}{[(p-k)^2+M^2F^4(p-k)]k^2}C_n^{(1)}\left(\frac{k\cdot p}{|k||p|}\right)|k|^n|p|^{2-n}\sim p^2\bar{\rho}^2\log(p\bar{\rho}).$$

where $C_n^{(1)}$ is Gegenbauer polynomial

Numerically one gets $b_{32} = -I(M) = -0.014$ at $M\bar{\rho} = 0.58$. From this we can make a rough estimate of the ratio

$$\frac{\mathcal{M}_3[h_L^{tw3}]}{\mathcal{M}_3[h_L^{tw2}]} \sim 10^{-2}\,. \tag{23}$$

Let us note that in the bag model the corresponding ratio is about 10 times larger [17, 21].

3. To summarize, we have shown that the instanton vacuum with its inherent small parameter, $\bar{\rho}/\bar{R}$, implies a parametrical (and numerical) hierarchy of the spin–dependent twist–2 and –3 matrix elements: $h_L^{tw3} \ll h_L^{tw2}$. The same hierarchy was observed for twist–2 and –3 contributions to g_2 [15], which seems to be confirmed by recent measurements by E155 collaboration [6].

We would like to thank A.V. Efremov, K. Goeke, A.Kotzinian, P.V. Pobylitsa and C. Weiss for fruitfull discussions.

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